Comment to: Corrections to the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle

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In the paper [F. Nasseri, Phys. Lett. B 632 (2006) 151–154], F. Nasseri supposed that the value of the angular momentum for the Bohr's atom in the presence of the cosmic string is quantized in units of $\hbar$. Using this assumption it was obtained an incorrect expression for Bohr radius in this scenario. In this comment I want to point out that this assumption is not correct and present a corrected expression for the Bohr radius in this background.

In a recent paper F. Nasseri[1] analyzed the fine structure constant in the spacetime of a cosmic string from the generalized uncertainty principle. The author claims that the Bohr's radius in this system increases by a factor of order $\pi/4 \times 10^{-6}$. Such a result was obtained by the author considering that the stationary orbits in the spacetime of a cosmic string have an integer number of wavelengths in the interval from 0 to 2π. This is not correct once for stationary orbits we have

$$\oint \frac{dS}{\lambda} = n; \quad n = 1, 2, 3, ... \tag{1}$$

where S is the length of the orbit and λ the wavelength. As the metric of a cosmic string is given by

$$ds^2 = c^2dt^2 - dz^2 - d\rho^2 - \rho^2 d\varphi'^2, \tag{2}$$

where $\varphi' = \left(1 - \frac{4G\mu}{c^2}\right)\varphi$, which implies that φ' varies from 0 to $2\pi b$. Therefore $dS = \rho d\varphi'$, leads to a correction in the condition of quantization of the angular momentum. Thus, instead of $L_n = n\hbar$ as considered by the author in [1], we have $L_{n_{(b)}} = \frac{n}{b}\hbar$, where b is the deficit angle and is given by $b = 1 - \frac{4G\mu}{c^2}$[2]. With this consideration, the radius of the nth Bohr orbit of the hydrogen atom in the presence of a cosmic string becomes

$$\rho_n = \frac{4\pi\epsilon n^2\hbar^2}{me^2\left(1 - \frac{\pi}{4}\frac{G\mu}{c^2}\right)\left(1 - 4\frac{G\mu}{c^2}\right)^2}. \tag{3}$$

The eq.(17) of [1] is correct only for flat spacetime. In the presence of a cosmic string, the radius of the nth Bohr orbit of the hydrogen atom is given by

$$\hat{a}_B = \frac{4\pi\epsilon\hbar^2}{me^2\left(1 - \frac{\pi}{4}\frac{G\mu}{c^2}\right)\left(1 - 4\frac{G\mu}{c^2}\right)^2}. \tag{4}$$

In the absence of a cosmic string, the lowest orbit ($n = 1$) of the Bohr orbit has the following expression

$$a_B = \frac{4\pi\epsilon\hbar^2}{me^2} = 5.29 \times 10^{-11} m. \tag{5}$$

Thus combining eqs.(4) and (5), we obtain

$$\frac{a_B}{\hat{a}_B} = \left(1 - \frac{\pi}{4}\frac{G\mu}{c^2}\right)\left(1 - 4\frac{G\mu}{c^2}\right)^2. \tag{6}$$

In the weak field approximation, eq.(6) turns into

$$\frac{a_B}{\hat{a}_B} \approx 1 - \frac{G\mu}{c^2}\left(8 + \frac{\pi}{4}\right). \tag{7}$$

In the limit $\mu \to 0$, i.e., in the absence of a cosmic string, $\frac{a_B}{\hat{a}_B} \to 1$. It is worth noticing that there is a difference between eq.(22) of 1 and eq.(6) of this Comment. Taking into account the correction given by eq.(6) and inserting $\frac{G\mu}{c^2} \simeq 10^{-6}$ we get

$$\hat{a}_B = \frac{a_B}{\left(1 - \left(8 + \frac{\pi}{4}\right) \times 10^{-6}\right)}. \tag{8}$$

From the above equation, we conclude that the numerical factor which correct the Bohr radius is different from the one obtained in eq.(23) of reference [1].

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[1] Forough Nasseri. Phys. Lett. **B632** (2006) 151.

[2] A. Vilenkin, E. P. S. Shellard, Cosmic string and Other

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